Article

Income-Based Segregation in Urban Areas: Framework for a Complex-Systems Approach

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Abstract
Income-based segregation is a fundamentally complex phenomenon. Though it can be analytically studied, developing an agent-based model allows us to study segregation at several levels of aggregation – to better understand the interactions that take place in between the micro- and macro- levels that lead to the pattern of segregation observed in modern cities. The spatial agent-based model of the housing market developed in this paper gives a foundation which can be built upon to tackle some of the key problems present in existing literature. We test the model to find that the degree to which households are willing to spend on rent relative to other expenditures and the extremity of variation of this degree in the population give rise to different patterns of segregation observed in the housing market. We assume a rental-only market, and that agents' incomes are locally spent. A section on guidance for further development and extension of the model is then presented.

Keywords: Income Segregation; Agent-Based Model; Housing Market; Urban Segregation; Complex System; Spatial Autocorrelation, Computation

1. Introduction

Any system for the organisation of social beings must inherently describe the networks they can access and influence. In the study of people, whether in an economic, political, or moral context, understanding their “topology” gives a causal notion of how the constituent parts of the system influence their networks: a critical intuition for macro behaviour. Cities lend themselves to this type of disaggregation, owing to the myriad people, interdependencies, and interactions they comprise. Their behaviour is characteristic of a complex economic system, defined here as a model of an economic phenomenon that builds up from constituent parts, is simulated through computation, and perpetually asks how people change their behaviour in reaction to the behaviour of others (Arthur, 2013). Income-based segregation is a commonplace result of the complex interactions that make up a city, with consequences ranging from exacerbated income inequality (Bailey et al., 2020) to the polarisation of ideas (Morales et al., 2019).

Existing research building an agent-based model of income segregation in cities is sparse. Prior to Schelling’s dynamic model of racial segregation (1971), income segregation models used equilibrium analysis. Schelling’s work, which sets out a grid-based model for residential racial segregation, has set standards in the field that spawned much further research in racial segregation. These standards have been applied to models of income segregation with some success, but gaps still exist in terms of minimising the number of assumptions made and ensuring model design factors such as intrinsic attractiveness of locations are determined endogenously and not chosen arbitrarily by the modeller.

By building a simple grid model of the rental housing market in one city, we can adapt the model to understand the specific factors that contribute to the spatial segregation of citizens by income. As a test, we vary the degree of homogeneity of the marginal rate of substitution (MRS) between housing and non-housing expenditure of agents to study how the spatial distribution of agents changes, finding that the greater the extremity of variation of the MRS, the lower the observed segregation effect as measured by both quantitative and qualitative factors.

In the interest of disclosure, this paper is not complete. It does not intend to cement the notions of utility, maximisation, or constrained choice such that they can stand up to the same well-established ideas in analytical models in economics. Regarding the critical issues outlined by those analysing existing literature on agent-based models of segregation (Crooks et al., 2008), neither does this paper constitute a solution to them. Rather, this paper seeks to be a beginning point for the study of simulated housing markets – a framework using which the reader can conduct their own experiments as suits their interest. In that respect, a full replication of the code used to build the model presented can be found on GitHub via the link provided in the Appendix.

2. Literature Review

Patterns of segregation in cities are well documented, with much of the literature focusing on race and income. Income-based segregation is defined here as the grouping of households with similar income levels into the same geographical or social space (Reardon and Bischoff, 2011). This review attempts to organise the key ideas employed in modelling the emergence of segregation, analyse how these learnings are applied to income-based segregation, and identify areas that require further, or different, thinking.

It is worth mentioning that the analysis of income-based segregation is fairly unorganised in the field. While modern researchers follow well-trodden paths, their approaches are always novel and highly variable. Crooks et al. (2008) aptly summarise this phenomenon, declaring that “there are as many models as modellers”.

Despite that, common themes arise.

2.1. Traditional Models of Income-based Segregation

Analytical models of income-based segregation mainly take root from Tiebout (1956), on the premise that households’ location choices are analogous to decisions between bundles of public goods available at each location. While his analysis concerns political affiliation, the principles of the model are nonetheless applicable to income-based segregation. As analytically demonstrated by Epple and Platt (1998), when heterogeneous households vote within their jurisdiction on taxes and expenditure and are free to move between jurisdictions that offer different tax-expenditure rates, an equilibrium state of partial income segregation arises. New research goes further, highlighting that income inequality – implicitly present in Epple and Platt’s model – creates market pressure toward segregation by incomes, since houses are chosen based on the attributes of

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the neighbourhoods they are in and the physical characteristics of the housing stock (Watson, 2006). Reardon and Bischoff (2011) verify this empirically, showing a positive correlation between income inequality and income-based segregation.

2.2. The Benefits of an Agent-based Approach

There are two reasons why we may favour a computational ABM approach. First, income-based segregation perpetuates its own cause: increased segregation can isolate social networks and increase inequalities between regions, leading to further pressure toward segregation (Schelling, 1969) and positive and negative feedback loops. While analytical approaches deal well with stationary points that arise from these loops, their usefulness is limited to these cases only, and falls short in describing the journey to these points and the timescales on which they may arise. Second, analytical models have to encode most of the “processing” work into the same space as the result they are depicting. Their graphs have to be incredibly information-dense, yet still show only a slice of the dynamics taking place.

In order to observe the feedback loops and document the interactions that individuals have within their city, agent-based models (ABMs) can be used. ABMs build a bottom-up model of the market, ensuring the micro-level interaction between individual actors is well-defined, allowing macro behaviour to emerge naturally, preserving heterogeneity, and removing the need for aggregation that masks the complexity in the transition from the micro to the macro level. Households are represented by singular agents with heterogeneous incomes, preferences, and behaviours. Their interactions are governed by common rules. Through simulation, segregation emerges endogenously and without any assumption of equilibrium. Arthur (2013) asserts that such thinking is of profound use to modelling systems with a multitude of interactions since the model takes equilibrium to be a special case of a more general non-equilibrium state that the system is in.

2.3. The Schelling Model and its Derivatives

The first such ABM applied to segregation in Schelling’s seminal work (Schelling, 1971) details racial segregation, but in principle, his model can act as a guide for desirable characteristics in ABMs of any kind of segregation. Schelling constructs a grid of houses that he populates with Black or White households, leaving some vacant. An individual household dissatisfied with the ethnic ratio in their local neighbourhood can move to the nearest vacant house. The model can be run by hand until a stable equilibrium is reached, where every actor is satisfied with their current position and no more moves can be made. For example, complete segregation would be a stable equilibrium, since a move to the opposite neighbourhood will cause dissatisfaction. Schelling demonstrates that even a small preference for a local majority in terms of race can lead to total segregation over time. Many authors have since developed Schelling’s model, remaining in the context of racial segregation (Spaiser et al., 2018; Barr and Tassier, 2008; Zhang, 2011; Grauwin et al., 2012). Promising research conducted by Bonakdar (2019) imposes housing, credit, and labour market constraints on Schelling’s model, finding that lock-in effects arise for low-income households which are also stuck with negative equity, since the price of favoured houses exceeds the agent’s income, especially when access to credit is low, though he does not investigate what lock-in effects imply for segregation. Bonakdar concludes that racial segregation diminishes in comparison to income segregation when such constraints are imposed and socioeconomic factors like income and education are non-zero.

2.4. ABMs of Income-based Segregation

While such market constraints are commonly applied to Schelling’s model, the literature on the direct application of ABMs to income-based segregation is comparatively sparse. Of the few papers that document the creation of such ABMs, the models laid out in Pangallo et al. (2019) and Gauvin et al. (2013) are the most developed, with the housing market forming their central context.

Pangallo et al. (2019), though building on the model set out by Gauvin et al. (2013), lay out a more general framework. The city created is grid-based but departs from Schelling’s by introducing an external “reservoir” of buyers, with internal movement not explicitly defined. Location attractiveness depends on both a static exogenous intrinsic term and on an endogenously determined term, which varies with the income statistics of the local neighbourhood. In both papers, this intrinsic attractiveness decreases with distance from the centre, therefore accounting for the “monocentric” concentration of amenities and job opportunities observed empirically. Pangallo et al. find their model agrees with empirical data (Reardon and Bischoff, 2011) and that positive feedback results in richer households locating closer to the centre. Gauvin et al. take a further step by adapting the model to Paris house prices in 1994, arranging for each district in Paris to be assigned its own intrinsic attractiveness, and find that their results agree with empirical observation.

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However, there exists a critical problem in using intrinsic attractiveness as a basis for agents' decisions. Since it is not endogenously determined, it reflects an assumption of the modeller of a typical city structure and in whatever way it is assigned, it imposes a condition on the model that is not necessary. Central business districts are not inherent characteristics of a city, rather they are creations of the actors within the city itself. That intrinsic attractiveness takes on certain values implicitly holds factors of amenities and opportunities constant in the area, when in fact, we have seen from Tiebout (1956) and Epple and Platt (1998) that consumer-voters choose their level of amenities through location, creating demand for them and boosting their production by picking higher tax-expenditure levels.

Gauvin adds depth to the issue through the introduction of a social influence term that characterises the degree to which incoming transactions at a location put upward pressure on attractiveness due to a higher intensity of demand at that location. When social influence is too low, heterogeneous attractiveness levels alone do not result in segregation. Yet, framing income segregation as a problem of social preferences for neighbours with particular incomes – a notion borrowed from Schelling's model (1971) – and not an issue of the direct effects of incomes on the quality of neighbourhood characteristics in itself cuts out key determinants of house prices, especially in a city, such as the local spending pattern of households. Furthermore, since these incomes are constant in both models, social mobility, defined as agents being able to change their place within the income distribution, is not accounted for.

3. Methodology

3.1. Defining the Objects

We now develop an agent-based computational model of a general city.

3.1.1. Agents

The model contains N agents, with each agent $n = 1, 2, \ldots, N$ earning a fixed heterogeneous income level $Y = y$ in each period T, where Y is a random variable drawn from a specified distribution common to the entire population that we can vary. Furthermore, each agent faces a trade-off between income spent on housing in period T and income toward non-housing expenditure, represented by the marginal rate of substitution (MRS) α between housing and non-housing expenditure.

Therefore,

$$y = h + d$$

where h is housing expenditure and d is non-housing expenditure.

$$h(y, \alpha) = \frac{y}{1+\alpha}, \qquad d(y, \alpha) = \frac{\alpha y}{1+\alpha}, \qquad \alpha > 0.$$

The MRS α is exogenous to the model; similar to the income level, it is a random variable drawn from a specified distribution that is in our control.

3.1.2 Houses

In order to preserve the elegance of Schelling's (1971) model, this paper considers an $M = M_1 \times M_2$ grid of houses (Figure 2), setting $M_1 = M_2$ without loss of generality.

381	69	333
348	5	225
166	312	182

Figure 1: The Moore neighbourhood (Gardner, 1970) of the house occupied by agent 5

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The utility $u_{(a,b)}$ of a house at position (a, b) is given by the simple mean of the incomes of the K agents occupying its Moore neighbourhood (Figure 1):

$$u_{(a,b)} = \sum_{k=1}^{K} \frac{y}{k}$$

This utility can be interpreted as the availabilities of local amenities (including public services such as schools and hospitals, and private facilities such as commercial areas) in line with Tiebout's (1956) theory that the choice of consumer-voters on the community they reside in is dependent on their preferences for local amenities.

It is important to note that $u_{(a,b)}$ can be easily modified to reflect how we want the incomes of the surrounding neighbourhood to translate into the utility of each house.

3.2. The Market Matching Process

Agents are initially allocated random houses in period $T = 0$, resulting in a grid similar to Figure 2.

171	139	97	73	169	328	221	29	312	263	61	57	116	340	175	79	244	195	65	217
121	245	301	14	98	337	192	99	310	397	161	267	47	141	115	56	202	365	222	39
395	70	26	67	288	64	227	232	194	317	11	109	339	396	82	30	381	106	149	201
302	230	173	219	274	394	130	318	358	140	373	156	315	117	329	200	28	268	74	177
265	7	93	386	382	186	17	314	183	363	167	95	342	212	252	388	137	362	110	290
162	248	66	27	19	258	31	160	205	198	170	58	83	147	251	380	25	111	293	389
319	105	85	352	123	81	126	112	100	273	132	20	62	305	104	2	398	357	75	145
283	220	101	266	118	375	125	238	260	6	239	187	68	12	181	134	182	114	46	338
354	343	34	355	378	119	240	23	253	191	313	285	166	275	131	150	51	190	377	60
231	193	284	286	215	351	96	306	179	180	211	153	21	35	282	103	196	270	87	399
40	323	254	33	204	371	168	127	48	348	350	289	299	164	197	250	393	142	135	38
155	72	59	138	281	259	146	158	88	346	133	279	55	242	151	49	353	347	10	262
264	8	255	22	152	3	53	326	243	159	157	1	90	308	269	272	367	44	189	316
368	13	207	24	276	307	178	309	384	246	41	226	208	344	76	203	84	124	278	294
91	249	185	206	128	224	223	37	297	144	18	32	277	257	15	210	311	387	9	325
391	92	4	233	364	148	122	129	280	236	360	256	379	36	235	291	94	374	229	199
172	361	218	69	345	341	334	369	370	241	300	136	108	143	5	356	216	321	332	385
50	89	390	184	296	247	372	336	383	292	237	324	359	392	349	113	327	320	78	214
43	376	330	213	16	63	228	45	366	154	331	400	295	107	271	225	298	174	54	42
335	176	86	77	209	322	163	165	52	287	261	333	234	304	102	303	188	120	80	71

Figure 2: A grid of size N= 20 X 20. Darker shades indicate agents with lower income

Each next period starts with a round of bidding, where every agent places a bid B on every house in the city, given by:

$$B = \frac{u_{(a,b)}}{u_{(a*,b*)}} \times h(y, \alpha)$$

where $(a*, b*)$ is the location of the house with the highest utility. Therefore, each agent prepares a bid for every house in order of preference based on housing utility $u_{(a,b)}$ so that in the event that they are outbid for their most preferred house, they have a next-best alternative.

All houses are then auctioned in a Dutch-style auction, starting with the highest bidder on each house. Once the new tenants are set, the utilities of all houses are updated with the incomes of new neighbours in preparation for the next round of bidding.

3.3. Key Assumptions of the Model

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i.) The market presented is purely rental, with tenancy contracts all lasting one period, after which they expire en masse. There is one common implicit "landlord" for all tenants in the city.

ii.) The city is assumed to be closed, with each agent able to occupy one household such that every house is occupied in every period. The landlord therefore sees demand for city housing only originating from the existing city population. No external reservoir of agents exists.

iii.) That the presence of local amenities is dependent on neighbouring incomes is a strong assumption; the implication is that income is locally spent, either through tax-based distribution, or through consumption of goods and services in local markets. The exact interaction between local incomes and amenities is coded for in $u_{(a,b)}$.

iv.) All actors have perfect information and well-defined choices – hence the market clears.

4. Results

To quantify the degree to which the city is segregated by income, we can use Moran's I (Moran, 1950), a metric of spatial autocorrelation. Moran's I is given by:

$$I = \frac{N}{\Sigma_i \Sigma_j w_{ij}} \frac{\Sigma_i \Sigma_j w_{ij}(y_i - \underline{y})(y_j - \underline{y})}{\Sigma_i (y_i - \underline{y})^2}$$

where i and j index a pair of houses, y_i and y_j are the incomes of the occupants of houses i and j, w_{ij} is a component of a spatial weights matrix that takes on 1 if houses i and j are neighbours and 0 if they are not, and $\underline{y}$ is the average income of the population. Moran's I takes values $-1 < I < +1$, with $I = -1$ indicating perfect dispersal analogous to a checkerboard pattern, $I = 0$ indicating random arrangement, and $I = +1$ implying perfectly segregated clusters.

Below are two typical evolutions of the model under various initial conditions.

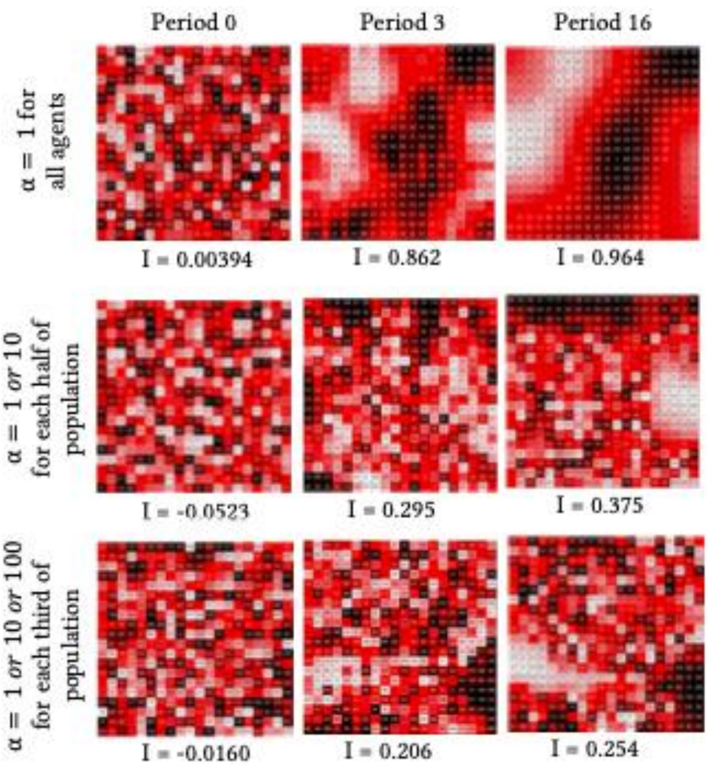

Figure 3: Varying with a uniform income distribution, N = 400, income range: [1000, 10000]

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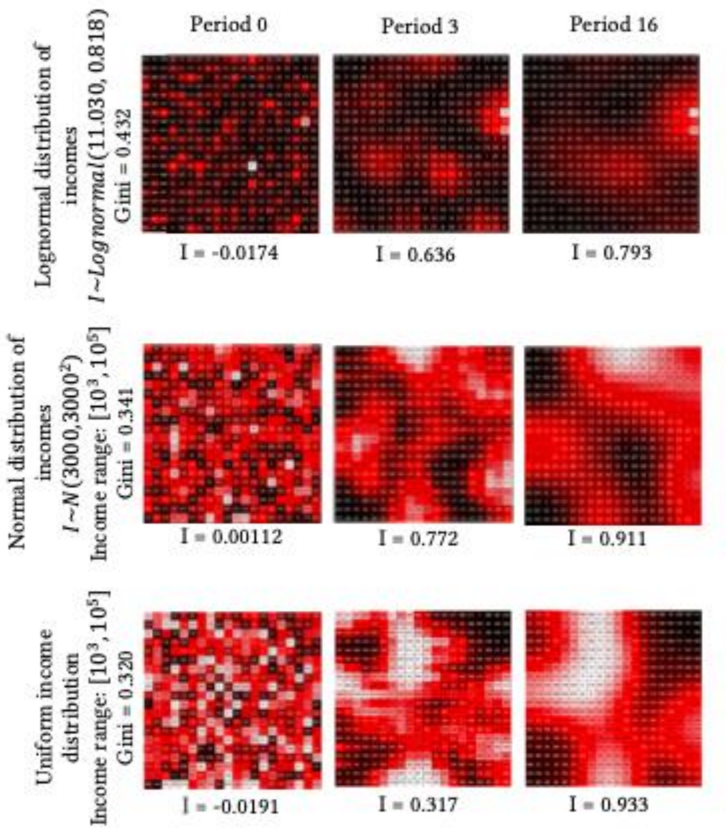

Figure 4: Varying income distribution with homogeneous α, N=400. Values for μ and σ in the lognormal distribution were obtained based on (Schield, 2018)

From these preliminary results, we can ascertain that when there is a more pronounced difference in willingness to pay for local amenities – as characterised by the presence of individuals with $\alpha = 1, 10,$ and 100 in the population – the Moran's I value reached after a number of periods falls, indicating a fall in income-segregation. This is similar to varying the income distribution but allows for more detailed interaction between agents' income and expenditure to be encoded using the MRS in future development.

5. Discussion and Points for Development

5.1. The MRS

What is the intuition behind the MRS and its impact on segregation? An individual agent, despite recognising the superior amenities available at one location over another, will still only allow a certain proportion $h = \frac{y}{1+\alpha}$ of their income to be spent on housing. Therefore, a rich agent with a high value of α places a similar bid on a given house to a poorer agent with a low value of α; the presence of individuals who do not place much importance on rental spending relative to other spending both *delays* and *reduces* the overall segregation effect.

However, the assumptions exclude both wealth effects of home ownership and social mobility. The introduction of social mobility is especially complicated, requiring either assumptions regarding (or the direct generation of) labour markets, education networks, and even technological development.

5.2. On Local Expenditure

Furthermore, these results rely on the implicit assumption that local income is locally spent – if only in part then in a consistent proportion across the whole city. Again, such an assumption cannot be removed without explicitly defining the avenues through which income is generated and spent. Here, a more efficient model structure using multiple mathematical graphs that describe the interdependencies between agents in different capacities can be employed. The spatial weights matrix w, which currently

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is only used to describe relationships between neighbours, can be replicated to account for other relationships, such as employment contracts, transport links, and goods and services trade, all of which play a prominent role in the housing decision.

5.3. What Do We Want Our Cities to Look Like?

Analysis of ONS (Office of National Statistics) data indicated that urban towns and cities in the UK see a value of Moran's I between -0.15 and 0.7 (ONS, 2021). Research on Facebook data conducted by Bailey et. al (2020) illustrates a link between socio-economic connectedness and social mobility – the networks people have access to can affect their maximum potential earnings in their life, suggesting two-way causality between income inequality and income-based segregation.

Just as income inequality naturally questions the degree of inequality that is tolerable – or even desirable – in society, policymakers here must make a judgement: how much segregation is too much? Adding social mobility to the presented model can allow us to study the origins of the ONS's and Bailey's findings, and the conditions under which they are mitigated, giving modellers the ability to recommend not just methods to achieve a given level of equity and mobility in society, but that level which is consistent with the behaviour of free people itself.

6. Conclusion

This paper has provided a review of the literature on modelling segregation, which was revolutionised by Shelling's (1969) paper that analysed racial segregation, and, upon considering the existing attempts to translate his model into a computational study of income-based segregation, has given a general, accessible framework that can be used for further research.

The grid-based model presented looks at the housing market in a closed city with one landlord and N agents with varying incomes and expenditure preferences occupying N households. By measuring the degree of segregation using the Moran's I value reached after a given number of periods, we show that the presence of citizens with varying propensities to spend on housing relative to other expenditures has a direct impact on the observed segregation pattern. The more variable and extreme the differences in the proportion of income spent on housing by each agent relative to another agent, the closer the Moran's I value reached was to 0, implying a lower degree of income segregation.

The eradication of certain assumptions, such as that local income is locally spent, and that incomes remain fixed for the whole simulation, can give rise to richer analysis. A model without these assumptions, we recommend, should replicate the spatial weights matrix defined for Moran's I to describe networks pertaining to the labour market, goods and services trade, and transportation to analyse income-based segregation more efficiently and with greater detail.

There are a number of further considerations which could be addressed in future research:

i.) *Challenges in geo-spatial ABMs*
Crooks et al. (2008) outline seven key challenges faced by modellers when creating ABMs. No widely accepted theoretical framework exists as with analytical models concerning utility, rents, rules for interaction, and other modelling standards. In Crooks (2010), he also proposes a non-grid-based model for a city, better capturing the high population densities present in modern metropolises.

ii.) *Matching ABMs with data*
A model is only as useful as it is applicable. New measures such as rank-order segregation proposed by Reardon et al. (2012) and the spatial ordering index proposed by Dawkins (2007) allow model data to be more accurately compared with empirical data so as to isolate the analysis of income segregation from the underlying income distribution.

iii.) *Artificial learning*
Though the discussed models are dynamic, the agents themselves are static beings. Incorporating ideas from Holland and Miller (1991) and Axelrod (1997) on agents adapting to novel circumstances through intelligent learning and genetic selection processes could add a new layer of complexity to the entire model.

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Appendix

Code for the simulation (Python 3.8, Windows 10)

Link: https://github.com/claudelorrain/Income-Segregation