Article

A Modal-Epistemic Framework of Decision-Making under Knightian Uncertainty

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Abstract
Drawing from Knight's original insight that uncertainty stems from the limits of knowledge, this paper presents a novel framework for modelling decision-making under Knightian uncertainty by integrating epistemic logic with the Choquet expected utility model. The framework formalises the agent's internal informational constraints as a set of epistemically accessible possible worlds, ranked by a plausibility order. From this structure, preferences are derived and evaluated using the Choquet integral. This approach allows ambiguity aversion to emerge endogenously as a rational response to incomplete knowledge, rather than a separate axiom. The proposed model identifies the agent's optimal choice that maximises expected utility given their epistemic structure, unifying economic and logical treatments of uncertainty. Applications are explored in innovation strategy and game theory. Despite empirical limitations, it offers a rigorous foundation for further research into decision-making under uncertainty, reinterpreting Knightian uncertainty through a formal epistemic structure.

Keywords: Decision Theory, Knightian Uncertainty, Modal Logic, Epistemic Logic

'We can foresee only what we ourselves construct.'

Ludwig Wittgenstein, Tractatus Logico-Philosophicus

1. Introduction

Uncertainty has long been a central theme in economics, famously articulated by Frank Knight in his 1921 work, *Risk, Uncertainty and Profit*. Knight drew a foundational distinction between risk, where probabilities are known, and uncertainty, where they are unknowable. While this distinction remains conceptually powerful, it is challenging to formalise Knightian uncertainty as conventional modelling tools rely on well-specified probability measures.

The aim of this paper is to clarify and preserve the spirit of Knight's concept, not by further axiomatising it, but by grounding it in a more fundamental, epistemic structure. While acknowledging the significance of existing frameworks, I propose a novel framework that reinterprets uncertainty as a consequence of epistemic limitation - the inherent constraints on what an agent can know. By integrating the framework of epistemic logic, a branch of modal logic used to model knowledge and possibility, with decision-theoretic tools, the paper sketches a new epistemic-plausibility model. In this model, the agent's knowledge defines a set of accessible worlds ranked by plausibility order. This order encodes the agent's epistemic stance without assigning precise probabilities, reflecting the qualitative nature of the uncertainty. Then, the plausibility ordering is transformed into a capacity, which allows the computation of Choquet integrals to rank acts. By combining the qualitative with the quantitative layers, the proposed model offers a clear decision rule when probability is not available or well-defined.

The paper proceeds as follows. Section 2 provides a literature review on Knightian uncertainty and discusses how other papers model uncertainty. Section 3 develops the epistemic-plausibility model in detail, formalising the relationship between an agent's knowledge, their plausibility judgments, and the Choquet integral. Section 4 discusses the implications, applications, and limitations of this approach, while Section 5 concludes by summarising the contributions and outlining avenues for future research.

2. Literature Review

2.1. Distinction

In Knight (1921)'s original account, risk is a type of 'measurable' uncertainty, and uncertainty is an 'unmeasurable' one (p. 20). If we interpret measurability as knowability, then this distinction is fundamentally epistemic, rooted in the limits of knowledge rather than the nature of the world itself. Knight's suggestion that understanding uncertainty requires an inquiry into the nature of knowledge (p. 199) also supports my reading.

Despite its intuitive appeal, Knight's classification contains conceptual ambiguity. As O'Donnell (2021) points out, defining uncertainty as 'unmeasurable uncertainty' is either tautological or contradictory unless the terms are carefully distinguished. To clarify Knight's account, Vercelli (1999) introduces the distinction between first- and second-order uncertainty: while the former refers to probabilistic attributes of phenomena, the latter concerns the measure of those probabilistic attributes. In this framing, Knight's 'unmeasurable uncertainty' can be understood as second-order uncertainty – a lack of clarity about the probabilities themselves. However, this raises a further question: if all forms of uncertainty (first- or second-order) reduce to probabilistic reasoning, is uncertainty simply another name for probability under epistemic constraints?

Knight anticipates this concern. Adapting from the statistical framework of Irving Fisher, Knight adopts a tripartite model of probability:

1) A priori probability, derived from known symmetries or logical structure.
2) Statistical probability, derived from empirical frequencies.
3) Estimates, which are subjective judgments made when no valid basis for calculation exists.

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The first two categories constitute objective probability, and situations governed by them are conditions of risk. Here, agents can deduce or induce the likelihood of outcomes. By contrast, under uncertainty, agents act on estimates, making a probability-like judgement in the absence of objective grounds under logical or empirical justification (Knight, 1921, p. 223).

An epistemic reading of Knight's understanding of probability aligns with his grounds: probability is not an ontological feature of the world but a response to partial knowledge. Thus, if we possessed perfect knowledge and the world were always certain, the concept of probability would be meaningless.

Under risk, where information is incomplete but sufficient to deduce or induce objective probability, the law of causality – itself an unverifiable assumption – guides reasoning. Under uncertainty, however, Knight asserts that under uncertainty, people know 'in a positive sense that there is no reason' (p. 222) and act as if there were. In other words, people guess a subjective probability of an event, hoping to approximate the numerical objective probability that would be known with sufficient information.

Though Knight himself never formalised a theory of subjective probability, his insights anticipated what later theorists would attempt to capture mathematically.

2.2. Literature on Formalising Uncertainty

A rigorous, axiomatic representation of decision-making under uncertainty was developed by Savage (1954). His framework assumes rational behaviour (complete and transitive preferences) of the agents, who act as if they are maximising subjective expected utility (SEU) with respect to a subjective probability distribution over a state-outcome space.

Among Savage's axioms, the sure-thing principle (STP) is particularly notable. To illustrate, consider a world S containing subsets A, B, C …, each denoting an event. s denotes a possible state in A. A rational decision-maker would then act according to the possible states of the world and the consequences associated with those states. Formally, individual consequences f, g, h… are contained in the set of all consequences F. The function $f(s)$ is the act, attaching the consequence to the state. In other words, an individual act is always state-outcome dependent.

The STP states that $f \preccurlyeq g$ given A if and only if for every $f' \preccurlyeq g'$, f agrees with $f'$and g agrees with g' in A, and f agrees with g and f' agrees with g in $\sim A$. In other words, if agents choose the same actions regardless of whether A happens or not, the uncertainty about the occurrence of A should not change their choices. Savage's representation theorem shows that if an agent's preferences satisfy the STP and other rationality axioms, then there exists a unique subjective probability function over states and a utility function over outcomes such that the agent acts as if they are maximising expected utility with respect to these.

Anscombe and Aumann's 'horse-race and roulette' model (1963) improves on Savage's SEU model, allowing for a more flexible treatment of uncertainty. It treats cases where one deals with both risk and uncertainty, combining subjective speculation and objective lotteries.

The STP, central to SEU, assumes that under uncertainty, people can evaluate exact consequences of each act in every possible state. This assumption is famously challenged by Ellsberg (1961), who presents scenarios where agents consistently prefer acting with known risks over ambiguity, where probabilities are unknown or ill-defined. If the STP holds, the agent is assumed to form consistent probability judgements over all possible states, even those with ambiguous information. Yet they display systematic ambiguity aversion, avoiding choosing outcomes involving unknown probabilities. Therefore, preferences under known probabilities do not match preferences under ambiguity, violating the STP.

To address the Ellsberg Paradox, Gilboa and Schmeidler (1994) propose the Choquet expected utility (CEU) model, which allows for a more flexible measure than the assumption of additive probabilities. The CEU model replaces

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additive probability measures with the non-additive fuzzy measure ν. The Choquet integral evaluates an act not by the expected value under a probability measure, but with decision weights according to their rank and capacity ν. ν could also be sued to reflect different attitudes towards ambiguity are reflected by the behaviour of (Beißner and Werner, 2023). For example, if ν is convex such that

$$\nu(A \cup B) + \nu(A \cap B) \geq \nu(A) + \nu(B),$$

In addition to Schmeidler (1989) with Choquet Expected Utility, much work in economics has also sought to capture situations of unknown probabilities, often referred to as ambiguity. Classic contributions include Gilboa and Schmeidler (1989) with the max–min expected utility model and Epstein and Schneider (1999) on recursive multiple priors. They provide rigorous ways of quantifying decision-making without precise probabilistic information. In macroeconomics, Hansen and Sargent (2008) develop robust control methods to capture decision-making under model misspecification. In finance, Epstein and Schneider (2010) investigate learning and portfolio choice under ambiguous signals. In decision theory, Klibanoff et al. (2005) propose the smooth ambiguity model, allowing attitudes toward ambiguity to be explicitly parameterised. These works demonstrate the variety of quantitative tools now available for formalising uncertainty.

While the above methods model behaviour under uncertainty, I worry that they may represent too significant a departure from Knight's original conception of uncertainty. In Knight's view, uncertainty is fundamentally an epistemic limitation — a certain event is seen as possible if it can be grouped with past known types (Knight, 1921, p. 247). Knight here gestures towards a modal understanding of probability insofar as it represents the accessibility of similar past events or states.

The formal foundations for modelling such epistemic constraints were laid by Hintikka(1962), which introduced a modal logic of knowledge and belief. Building on Hintikka's work, epistemic logic has since been selectively applied in economics, particularly in the analysis of information and belief structures. Fagin and Halpern (1990) and Fagin et al. (1995) model knowledge and common knowledge using Kripke semantics, with applications to game theory and interactive reasoning. Aumann (1999) and Bonanno (2008) further develop epistemic frameworks for understanding strategic behaviour under information constraints. These models clarify the structure of information and reasoning but are rarely integrated with formal models of economic uncertainty.

In the next section, I will present a framework that takes into consideration agents' epistemic constraints and structures what they can know, believe, and prefer across possible worlds under uncertainty. To my knowledge, this is the first attempt to embed Knightian uncertainty within a unified modal–epistemic and decision-theoretic framework. I believe it can serve as a more faithful formalisation of Knightian uncertainty.

3. Modal-Epistemic Evaluation under Uncertainty

I believe that decision-making under uncertainty is fundamentally epistemic: agents act under epistemic constraints, having limited knowledge of the informational structure of the world. Current frameworks impose a series of externally defined functions following basic behavioural patterns, but decisions ultimately emerge from within an agent's subjective epistemic limitations. Therefore, I propose a more holistic setup which not only draws on standard tools from modal and epistemic logic (see Kripke, 1963; Hintikka, 1962; Fagin et al., 1995; Blackburn et al., 2001), but also extends them by embedding a plausibility-based capacity into the Choquet Expected Utility (CEU) framework.

For readers unfamiliar with modal logic and epistemic logic, Section 3.1 is a brief introduction to modal semantics, followed by Section 3.2, which outlines basic epistemic logic under modal semantics. In Section 3.3, a formal representation of the decision-making model is presented, integrating both the CEU model and epistemic logic.

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3.1. Modal Semantics

Modal logic is a type of *intensional* logic, a branch of formal logic that applies modality to qualify the truth of a statement. It lets us talk not only about what is true, but about what must be true (necessary) or could be true (possible). At the heart of the semantics lies the concept of *possible worlds*, an assumption that propositions are evaluated relative to a set of worlds, each representing an alternative way the world might have been.

The two standard modal operators are '□' (necessity) and '◇' (possibility). Let φ be a formula. In possible world semantics, we have:

Necessity $\Box\varphi$ is true if and only if φ is true in every possible world.

Possibility $\Diamond\varphi$ is true if and only if φ is true in at least one possible world.

Formally, modal logic can be framed under a Kripke model, defined as a tuple $\mathfrak{M} = (W, R, V)$, where:

- $W = \{w_1, w_2, \dots, w_n\}$ is a finite set of possible worlds.
- $R \subseteq W \times W$ is the accessibility relation on W, where wRv means world v is accessible from world w.
- $V: Prop \to P(W)$ is a valuation function assigning to each atomic proposition p the set of worlds in which p is true.

To put it in simple terms, 'necessary' = true in all the scenarios you currently regard as live, and 'possible' = true in at least one of those scenarios.

Example: Let p be 'unemployment < 5%'. If all scenarios you consider live have unemployment below 5%, then $\Box p$ holds. If at least one such scenario has unemployment below 5%, then $\Diamond p$ holds.

The satisfaction relation $M, w \vDash \varphi$ specifies when a formula φ is true at a world w in model M. It is evaluated recursively based on induction from the valuation of an atomic formula p, followed by evaluations of formulae with logical connectives and modal operators. Formally, for M at $w \in W$:

1. $M, w \vDash p$ iff $w \in V(p)$;
2. $M, w \vDash \neg\varphi$ iff $M, w \nvDash \varphi$;
3. $M, w \vDash \varphi \wedge \psi$ iff $M, w \vDash \varphi$ and $M, w \vDash \psi$;
4. $M, w \vDash \Box\varphi$ iff $M, v \vDash \varphi$ for all $v \in W$, wRv;
5. $M, w \vDash \Diamond\varphi$ iff $M, v \vDash$ for some $v \in W$, wRv.

Lastly, in modal logic, different systems are distinguished by the properties or assumptions imposed on the accessibility relation R. Two standard systems are **S4** and **S5**.

S4: ***R*** **is reflexive and transitive.**

Reflexive ($\forall w\ wRw$) - you never rule out the actual state.

Intuition: your information cannot exclude where you actually are.

Transitive ($\forall a, b, c \in W, aRb \wedge bRc \Rightarrow aRc$) - possibilities are stable under 'looking further'.

Intuition: if from a you think b could be the case, and from b you would still treat c as live, then from a you should already treat c as live.

S5: on top of S4, add symmetry.

Symmetric ($\forall a, b \in W, aRb \Rightarrow bRa$) – indistinguishability goes both ways.

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Intuition: if state a is observationally indistinguishable from b, then b is indistinguishable from a; information forms partition cells of mutually accessible states. **S5** thus makes R an equivalence relation (reflexive, symmetric, transitive).

3.2. Epistemic Logic

Epistemic logic extends modal logic to reason about what agents know or believe, rather than about necessity and possibility in general (Fagin et al., 1995).

The modal operator (analogous to '□' in classical modal logic)

- $K_a\varphi$: 'agent a knows that φ'

allow for the expression of states about an agent's knowledge or belief about certain propositions. This is useful in the context of decision-making under uncertainty.

In my framework, I adopt the modal system **S4** to characterise the epistemic accessibility relation. The relation R is assumed to be reflexive, such that the agent always considers the actual world epistemically possible. Transitivity ensures the agent's knowledge is consistent across chains of accessible worlds. Importantly, without assuming symmetry, this framework acknowledges that agents may learn new facts but would not be able to 'unlearn' any prior epistemic states.

3.3. Epistemic-Plausibility Model

In this section, I present a formal framework for decision-making under uncertainty. The model brings together epistemic logic, plausibility-based belief, and the Choquet expected utility.

Definition 1 (Language). Let Φ be a set of propositional variables with each variable φ representing an action-dependent outcome. The language $\mathcal{L}$ is defined by the syntax rule

$$\varphi ::= p \mid \neg\varphi \mid \varphi \wedge \psi \mid K_a\varphi$$

where $p \in \Phi$. Operator K_a denotes knowledge. It captures what the agent a knows under uncertainty, restricting decisions to epistemically possible worlds.

Definition 2 (Epistemic-plausibility model). An epistemic-plausibility model is a tuple

$$\mathfrak{M} = (W, R_K, \succcurlyeq, u, V)$$

in which

(i) $W \neq \emptyset$; stands for all possible states of the world;
(ii) $R_K \subseteq W \times W$; it is a reflexive and transitive epistemic accessibility relation, linking worlds the agent cannot distinguish due to limited knowledge;
(iii) $\succcurlyeq$ is a plausibility preorder over $A(w)$ (accessible worlds at w) ;
(iv) $U: W \to \mathbb{R}$; stands for the utility function, quantifying the preferences;
(v) $V: \Phi \to \mathcal{P}(w)$; stands for a valuation function for proposition φ.

R_K tells us which worlds we can see, $\succcurlyeq$ tells us how plausible each looks, U tells us how much we like them. For example, Let $W = \{H, M, L\}$ be knowledge of high/medium/low profit worlds. if we have $M \succ H \succ L$, then the agent thinks (and knows) medium profit is most plausible, followed by high and then low.

Definition 3 (Satisfaction relation). For a model $\mathfrak{M} = (W, R, \leq, U, V)$ and a world $w \in W$, the satisfaction relation M at $w \in W$ is given by induction on φ:

1. $M, w \vDash p$ iff $w \in V(p)$;
2. $M, w \vDash \neg\varphi$ iff $M, w \nvDash \varphi$;

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3. $M, w \vDash \varphi \wedge \psi$ iff $M, w \vDash \varphi$ and $M, w \vDash \psi$;
4. $M, w \vDash K_a\varphi$ iff $M, v \vDash \varphi$ for all $v \in [w]_R$;

where $A(w) = \{v|wRv\}$ denotes the set of epistemically accessible worlds from w.

Definition 4 (Capacity function). From $(A(w), \succcurlyeq)$, define a capacity ν on subsets of $A(w)$:

$$\nu(\varnothing) = 0, \quad \nu\big(A(w)\big) = 1$$

and, for all $E \subseteq A(w)$:

$$\nu(E) = \max_{\omega \in E} \pi(\omega),$$

where $\pi: A(w) \to [0,1]$ is a plausibility scale preserving $\succcurlyeq$ (i.e., if $w' \succcurlyeq w''$ then $\pi(w') \geq \pi(w'')$).

The intuition is the capacity function ν assignis to each nonempty $E \subseteq A(w)$ the plausibility score of its maximally plausible elements. It then turns an ordinal plausibility into a numeric weighting for events, which is the key to connecting logic to CEU.

Proposition 1. The capacity ν derived from the plausibility ordering $\succcurlyeq$ satisfies, for all $E, F \subseteq A(w)$:

1. **(Monotonicity)** If $E \subseteq F$, then $\nu(E) \leq \nu(F)$.
2. **(Convexity)** $\nu(E \cup F) + \nu(E \cap F) \geq \nu(E) + \nu(F)$.

Proof. Monotonicity: If $E \subseteq F$, then every element of E is also in F, so the most plausible element of E is weakly less plausible than that of F under $\succcurlyeq$. Since π preserves $\succcurlyeq$, we have $\nu(E) \leq \nu(F)$.
Convexity: Let $p_E = \nu(E)$ and $p_F = \nu(F)$. By construction, $\nu(E \cup F) = \max\{p_E, p_F\}$ and $\nu(E \cap F) \geq \min\{p_E, p_F\}$. Adding them gives:

$$\nu(E \cup F) + \nu(E \cap F) \geq \max\{p_E, p_F\} + \min\{p_E, p_F\} = p_E + p_F,$$

which is exactly the convexity condition.

Definition 5 (Choquet integral). Given $U_f: A(w) \to \mathbb{R}$, the Choquet value is:

$$C_\nu(U_f) = \sum_{k=1}^{n} \Big(U_f\big(w_{(k)}\big) - U_f\big(w_{(k-1)}\big)\Big)\nu(A_k),$$

where $\big(w_{(1)}, \dots, w_{(n)}\big)$ orders worlds in $A(w)$ by increasing U_f, and $A_k = \{w_{(k)}, w_{(k+1)}, \dots, w_{(n)}\}$ denotes tail sets, which the set of the k-th best world and all better ones. As the capacity function $\nu(A_k)$ is non-additive but a weight of that tail set, we cannot simply add things up but calculate the incremental gain in utility when moving from one rank to next.

Definition 6 (Preferences).

$$f \succcurlyeq g \quad \text{iff} \quad C_\nu(U_f) \geq C_\nu(U_g).$$

We prefer the act with the higher plausibility-weighted Choquet value.

Theorem 1 (Optimal decision rule). In the epistemic–plausibility model, the optimal act is any f^* satisfying $f^* \in \operatorname*{argmax}_f C_\nu(U_f)$, where C_ν is the Choquet integral with respect to ν over $A(w)$.

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Proof. By Definition 6, $f \succcurlyeq g$ iff $C_v(U_f) \geq C_v(U_g)$. The optimal act is therefore the f^* such that $C_v(U_{f^*}) \geq C_v(U_f)$ for all f. This is equivalent to $\text{argmax}_f C_v(U_f)$, exactly as in standard SEU where v is additive. The difference is that here v is derived from epistemic plausibility, not subjective probability, but the maximisation logic is identical.

4. Discussion

Existing literature usually follows either the quantitative method of Savage, explicating actions in a probability structure, or treats the issue as purely qualitative, as presented by most works addressing decision-making through modal logic. This paper bridges the gap, providing formal semantics as the foundation of further mathematical analysis.

4.1. Model Implications

The key theorem that the agent's optimal decision maximises CEU under a plausibility-based capacity has several important implications. First, it models uncertainty internally, contrasting with how traditional economic models treat uncertainty as an external failure: be it a breakdown in probabilistic information, a lack of priors, or a defect in the environment. Uncertainty, in this view, is derived negatively as 'non-risk'. By contrast, this model reconstructs uncertainty as a set of epistemic limitations, given by the agent's internal representation of the world. The shift towards an internal logic is analogous to that of Kant's 'Copernican Revolution': uncertainty is structured by the limits of cognition, and if we shall attempt to model it, it should be precisely modelled from within the agent's epistemic horizon.

Second, it shows ambiguity aversion as a rational epistemic response to incomplete knowledge. Conventionally, the observed behaviour of ambiguity aversion is postulated through a series of axioms, for example, the Choquet integral. However, the behaviour is not explained properly by the CEU model alone. Structuring the capacity function as a result of the agent's plausibility orderings grounds ambiguity aversion in the logic of epistemic constraints. The model now becomes explanatory. We can now see that the agent considers only epistemically accessible worlds and ranks them by plausibility. Then their preferences become a natural response to a lack of information – the ambiguity aversion is not a bias, but an epistemic caution.

Together, these features enhance explanatory depth, offering structural coherence with the assumption of rational behaviour.

4.2. Applications in Economics

One notable application of decision theory under uncertainty, as mentioned by Knight himself, is entrepreneurial action and innovation. Structural uncertainty is often faced by innovators. For them, the future state of their strategy is not fully known, and probabilities often cannot be derived due to a lack of statistical occurrences. In this case, firms or investors who make decisions can be modelled based on plausibility rankings of outcomes, derived from past knowledge and plausible technological paths. It is more realistic than assumed probabilities.

Another important application is game theory. Analyses in game theory typically focus on strategies based on information about each other's payoffs. But perfect information is seldom the case. Instead of considering mutual best responses, one could define equilibrium in terms of Choquet-dominated strategies within each agent's plausibility space. This explains why sometimes players avoid strategies that seem too risky, even if they would yield high payoffs under a probabilistic model. In addition to the modal approach to game theory discussed in several existing works (Bonanno, 2002; Roy, 2021), my model incorporates mathematical components, potentially making it easier to be applied in real-life cases.

4.3. Illustrative Example: Entrepreneur's Decision

Consider an entrepreneur deciding how much to invest when opening a restaurant. The set of epistemically possible worlds is:

$$A(w) = \{H, M, L\},$$

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where: H, M, L corresponds to High, Medium, and Low profit scenarios

We model three acts:

- f: Aggressive investment (e.g., large venue, premium interior, extensive marketing)
- g: Modest investment (e.g., smaller space, moderate marketing)
- h: No investment (safe option, no restaurant opened)

The utilities U_f, U_g, U_h represent the entrepreneur's payoff (in normalized monetary or utility units) in each profit scenario:

World	U_f	U_g	U_h
H	9.5	9.0	6.0
M	3.0	8.8	4.0
L	-2.0	5.0	2.0

When economy is booming and generates H high profits scenario, all investments pay off well. If the entrepreneur chooses to not open a restaurant and put the money somewhere else, it yields modest safe return (6.0). In M (medium profits), aggressive investment still beats no investment, but modest investment is better due to lower costs. In L (low profits), aggressive investment loses money (-2.0), modest investment still clears costs (5.0), and no investment yields a safe positive return (2.0).

Capacity function. Let ν be a non-additive capacity over subsets of $A(w)$, representing the entrepreneur's subjective evaluation of sets of profit outcomes.

$$\nu(\{L\}) = 0.25, \quad \nu(\{M\}) = 1.00, \quad \nu(\{H\}) = 0.60.$$

which translate to $M \succ H \succ L$. The entrepreneur is very confident about medium profits, moderately confident about high profits, and pessimistic about low profits, reflecting ambiguity attitudes.
For larger sets, ν is defined as:

$$\nu(S) = \max\{\nu(\{x\}): x \in S\}.$$

which means that, for example, when M and H are both possible state of word, M being more plausible with $\nu(\{M\}) = 1 > \nu(\{H\})$ leads to $\nu(\{M, H\}) = 1$. The capacity fucntion ν takes on the plausibility score of M instead of H as M is the most plausible

Choquet Integral. We now compute the Choquet integral $C_\nu(U_a)$ for each act $a \in \{f, g, h\}$. Recall that

$$C_\nu(U_f) = \sum_{k=1}^{n} \left(U_f(w_{(k)}) - U_f(w_{(k-1)})\right) \nu(A_k)$$

<u>**Act f: Aggressive investment**</u>

First, let's sort ascending by utility: $L(0.0)$, $M(5.0)$, $H(9.5)$. This means that we have $A_1 = \{L, M, H\}$, $A_2 = \{M, H\}$, and $A_3 = \{H\}$.

- $k = 1$: $\Delta U = U_f L - 0 = -2.0 - 0 = -2.0 \Rightarrow \nu(A_1) = \max(0.25, 1.00, 0.85) = 1.00 \Rightarrow -2.0 \times 1.00 = -2.$
- $k = 2$: $\Delta U = U_f M - U_f L = 3.0 - (-2.0) = 5.0 \Rightarrow \nu(A_2) = \max(1.00, 0.85) = 1.00 \Rightarrow 5.0 \times 1.00 = 5.0.$
- $k = 3$: $\Delta U = U_f H - U_f M = 9.5 - 3.0 = 6.5$, $A_3 = \{H\} \Rightarrow \nu(A_3) = 0.60 \Rightarrow 6.5 \times 0.6 = 3.90.$

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$$C_v(U_f) = -2 + 5 + 3.9 = \mathbf{6.9}$$

Act g: Modest investment

Similar to the calculation process above, we have utilities from different worlds in the ascending: $L(5.0)$, $M(8.8)$, $H(9.0)$.

- $k = 1$: $\Delta U = 5.0 - 0 = 5.0$, $A_1 = \{L, M, H\} \Rightarrow \nu(A_1) = 1.00 \Rightarrow 5.0 \times 1.00 = 5.0$.
- $k = 2$: $\Delta U = 8.8 - 5.0 = 3.8$, $A_2 = \{M, H\} \Rightarrow \nu(A_2) = 1.00 \Rightarrow 3.8 \times 1.00 = 3.8$.
- $k = 3$: $\Delta U = 9.0 - 8.8 = 0.2$, $A_3 = \{H\} \Rightarrow \nu(A_3) = 0.6 \Rightarrow 0.2 \times 0.6 = 0.12$.

$$C_v(U_g) = 5 + 3.8 + 0.12 = \mathbf{8.92}$$

Act h (no investment)

Sort ascending: $L(2.0)$, $M(4.0)$, $H(6.0)$.

$$k = 1: \quad \Delta U = 2.0 - 0 = 2.0,\ A_1 = \{L, M, H\} \Rightarrow \nu = 1.00 \Rightarrow 2.0.$$
$$k = 2: \quad \Delta U = 4.0 - 2.0 = 2,\ A_2 = \{M, H\} \Rightarrow \nu = 1.00 \Rightarrow 2.$$
$$k = 3: \quad \Delta U = 6.0 - 4.0 = 2,\ A_3 = \{H\} \Rightarrow \nu = 0.6 \Rightarrow 1.2.$$

$$C_v(U_h) = 2 + 2 + 1.2 = \mathbf{5.2}$$

Therefore, g wins and the entrepreneur should choose to invest moderately. The intuition that due to ambiguity aversion and M is the most plausible, g is the more optimal choice from highest utility $U_g(M)$. As the High profit scenario is not as plausible and negative utility if investing aggressively under Low profit scenario, you could see that act f is not much better than h.

4.4. Limitations and Future Research Directions

The theoretical beauty of this model may be promising, but several components raise concerns. Plausibility orders, for example, might not be computable and universal. The model uses non-observable mental constructs for plausibility order, so it is challenging to recover individual plausibility orders from observed choice data in applied economic settings. In practice, one would likely need to run laboratory experiments or controlled field trials to identify plausibility orders and validate the model. Furthermore, plausibility orders are shaped by subjective judgment. A black swan event like COVID-19, unanticipated by anyone, could render any existing plausibility order inaccurate.

As a static framework, it assumes a fixed set of possible worlds and relations. However, agents learn and adapt their beliefs over time. In a dynamic economic environment, it would be crucial to develop a framework with temporal components and belief updates to capture learning under uncertainty. Examples can be dynamic epistemic logic or epistemic updates via public announcement logic (Benthem and Liu, 2021).

The current model also treats epistemic structure as uniform across agents. In reality, agents differ in cognitive access to information and in epistemic confidence. Extending the framework to accommodate heterogeneity would better support multi-agent applications, including those in game theory.

5. Conclusion

The epistemic-plausibility model provides a novel formalisation of decision-making under Knightian uncertainty by grounding the agent's evaluative process in epistemic logic. The model is distinguished from both classical expected utility theory and the Savage framework by integrating the Choquet integral within a modal structure. Instead of an external probability distribution, the agent's decision is modelled as driven by an internal epistemic structure of accessible possible worlds and a plausibility ordering. This formalisation aligns with Knight's original insight that

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uncertainty arises from a lack of knowledge regarding the true states of the world. In this light, agents act through their ignorance and make decisions with reason shaped by their own constraints.

Beyond the theoretical contribution, this framework has potential applications in areas such as innovation and entrepreneurship, game theory, and policy analysis, where strategic choices must be made under ambiguity. Future extensions could incorporate dynamic belief updates and agent heterogeneity, broadening its relevance to complex, real-world decision contexts.

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References

Anscombe, F. J. & Aumann, R. J., 1963. A Definition of Subjective Probability. The Annals of Mathematical Statistics, 34(1), pp. 199-205. doi: 10.1214/aoms/1177704255.

Aumann, R. J. (1999), 'Interactive epistemology i: Knowledge', International Journal of Game Theory 28(3), 263–300. doi: 10.1007/s001820050111

Beißner, P. & Werner, J., 2023. Optimal Allocations with α-MaxMin Utilities, Choquet Expected Utilities, and Prospect Theory. Theoretical Economics, 18(3), pp. 993-1022. doi: 10.3982/TE5060.

Benthem, J. v. & Liu, F., 2021. Deontic Logic and Changing Preferences. In: D. Gabbay, et al. eds. Handbook of Deontic Logic and Normative Systems, Volume 2. s.l.:College Publications, pp. 307-364.

Blackburn, P., de Rijke, M. and Venema, Y. (2001), Modal Logic, Cambridge University Press, Cambridge, UK.

Bonanno, G., 2002. Modal logic and game theory: Two alternative approaches. Risk Decision and Policy, 7(03), pp. 309-324. doi: 10.1017/S1357530902000704.

Bonanno, G. (2008), Epistemic foundations of game theory, Routledge.

Ellsberg, D., 1961. Risk, Ambiguity, and the Savage Axioms. The Quarterly Journal of Economics, 75(4), pp. 643-669. doi: 10.2307/1884324.

Epstein, L. G. and Schneider, M. (1999), 'Recursive multiple-priors', Journal of Economic Theory 94(1), 1–31. doi:10.1016/S0022-0531(03)00097-8

Epstein, L. G. and Schneider, M. (2010), 'Ambiguity and asset markets', Annual Review of Financial Economics 2(1), 315–346.

Fagin, R. and Halpern, J. Y. (1990), 'A logic for reasoning about knowledge and probability', Information and Computation 87(1-2), 78–128. doi: 10.1016/0890-5401(90)90060-U

Faulkner, P., Feduzi, A., C. R. McCann, J. & Runde, J., 2021. F. H. Knight's Risk, Uncertainty, and Profit and J. M. Keynes' Treatise on Probability after 100 years. Cambridge Journal of Economics, Volume 45, pp. 857-882. doi: 10.1093/cje/beab035.

Gilboa, I. and Schmeidler, D. (1989), 'Maxmin expected utility with non-unique prior', Journal of Mathematical Economics 18(2), 141–153. doi: 10.1016/0304-4068(89)90018-9

Gilboa, I. & Schmeidler, D., 1994. Additive representations of non-additive measures and the Choquet integral. Annals of Operations Research, Volume 52, pp. 43-65. doi: 10.1007/BF02032160.

Hansen, L. P. and Sargent, T. J. (2008), Robustness, Princeton University Press.

Hintikka, J. (1962), Knowledge and Belief: An Introduction to the Logic of the Two Notions, Cornell University Press.

Klibanoff, P., Marinacci, M. and Mukerji, S. (2005), 'A smooth model of decision making under ambiguity', Econometrica 73(6), 1849–1892. doi: 10.1111/j.1468 0262.2005.00640.x

Knight, F. H., 1921. Risk, Uncertainty and Profit. Boston: Houghton Mifflin Company.

Kripke, S. A. (1963), 'Semantical considerations on modal logic', Acta Philosophica Fennica 16, 83–94.

O'Donnell, R., 2021. Keynes and Knight: risk-uncertainty distinctions, priority, coherence and change. Cambridge Journal of Economics, Volume 45, p. 1127–1144. doi: 10.1093/cje/beab034.

Roy, O., 2021. Deontic Logic and Game Theory. In: D. Gabbay, et al. eds. Handbook of Deontic Logic and Normative Systems, Volume 2. s.l.:College Publications, pp. 765-782.

Savage, L. J., 1972. The foundations of statistics. 2nd rev. ed ed. New York: Dover.

Schmeidler, D., 1989. Subjective Probability and Expected Utility without Additivity. Econometrica, 57(3), pp. 571-587. doi: 10.2307/1911053.

Vercelli, A., 1999. The Recent Advances in Decision Theory under Uncertainty: a Non-Technical Introduction. In: L. Luini, ed. Uncertain Decisions: Bridging Theory and Experiments. s.l.:Kluwer Academic Publishers, pp. 238-260.

https://doi.org/10.14324/111.444.2755-0877.2048